Archibald First School Calculation Policy

Children are introduced to the processes of calculation through practical, oral and mental activities. As children begin to understand the underlying ideas they develop ways of recording to support their thinking and calculation methods, use particular methods that apply to special cases, and learn to interpret and use the signs and symbols involved. Over time children learn how to use models and images, such as empty number lines, to support their mental and informal written methods of calculation. As children's mental methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and succinct and lead to efficient written methods that can be used more generally. When faced with a calculation, children are able to decide which method is most appropriate and have strategies to check its accuracy. Being able to use these written methods gives children an efficient set of tools they can use when they are unable to carry out the calculation in their heads or do not have access to a calculator. At Archibald First School we want children to know that they have such a reliable, written method to which they can turn when the need arises.

The overall aim is that when children leave Archibald First School they:

- have a secure knowledge of number facts and a good understanding of the four operations;
- are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and particular strategies to special cases involving bigger numbers;
- make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- have an efficient, reliable, compact written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;
- use a calculator effectively, using their mental skills to monitor the process, check the steps involved and decide if the numbers displayed make sense.
- decide when it is best to use a mental, written or calculator method based on the knowledge that they are in control of this choice, as they begin to carry out all three methods with confidence.

Mental methods of calculation

Oral and mental work in mathematics is essential, particularly so in calculation. Early practical, oral and mental work must lay the foundations by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts. Later work must ensure that children recognise how the operations relate to one another and how the rules and laws of arithmetic are to be used and applied. Ongoing oral and mental work provides practice and consolidation of these ideas. It must give children the opportunity to apply what they have learned to particular cases, exemplifying how the rules and laws work, and to general cases where children make decisions and choices for themselves.

The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined. A good knowledge of numbers or a 'feel' for numbers is the product of structured practice and repetition. It requires an understanding of number patterns and relationships developed through directed enquiry, use of models and images and the application of acquired number knowledge and skills. Secure mental calculation requires the ability to:

- recall key number facts instantly – for example, all addition and subtraction facts for each number to at least 10 (Year 1), sums and differences of multiples of 10 (Year 2) and multiplication facts up to 10 × 10 (Year 4);
- use taught strategies to work out the calculation – for example, recognise that addition can be done in any order and use this to add mentally a one-digit number or a multiple of 10 to a one-digit or two-digit number (Year 1), partition two-digit numbers in different ways including into multiples of ten and one and add the tens and ones separately and then recombine (Year 2), when applying mental methods in special cases (Year 5);
- understand how the rules and laws of arithmetic are used and applied – for example, to add or subtract mentally combinations of one-digit and two-digit numbers (Year 3), and to calculate mentally with whole numbers and decimals (Year 6).

Written methods of calculation

The 2006 Numeracy strategy set out progression in written methods of calculation that highlights how children would move from informal methods of recording to expanded methods that are staging posts to a compact written method for each of the four operations.

The aim is that by the end of year 4 (when the children leave Archibald First School), the great majority of children should be able to use an efficient written method for each operation with confidence and understanding. This guidance promotes the use of what are commonly known as 'standard' written methods – methods that are efficient and work for any calculations, including those that involve whole numbers or decimals. They are compact and consequently help children to keep track of their recorded steps. Being able to use these written methods gives children an efficient set of tools they can use when they are unable to carry out the calculation in their heads or do not have access to a calculator. We want children to know that they have such a reliable, written method to which they can turn when the need arises.

In setting out these aims, the intention is that schools adopt greater consistency in their approach to calculation that all teachers understand and towards which they work. There has been some confusion as to the progression to written methods and for too many children the staging posts along the way to the more compact method have instead become end points. While this may represent a significant achievement for some children, the great majority are entitled to learn how to use the most efficient methods. The challenge for teachers is determining when their children should move on to a refinement in the method and become confident and more efficient at written calculation.

Being a First School the incidence of children moving between schools and localities is very high. Moving to a school where the written method of calculation is unfamiliar and does not relate to that used in the previous school can slow the progress a child makes in mathematics. There will be differences in practices and approaches which can be beneficial to children. However, if the long-term aim is shared across all schools in the Gosforth pyramid and if expectations are consistent then children's progress will be enhanced rather than limited. At Archibald First School we have developed good links with our feeder schools and will aim to make the transition in calculation methods for our pupils a smooth one. The entitlement to be taught how to use efficient written methods of calculation is set out clearly in the renewed objectives.

Objectives

The objectives in the revised Framework show the progression in children's use of written methods of calculation in the strands 'Using and applying mathematics' and 'Calculating'. The school also has a "non-negotiables" policy which covers the areas of mathematical learning in

which the children are not expected to provide a written method of working out and are expected to ‘just know it’. These objectives are what were once considered “mental maths”.

Using and applying mathematics	**Calculating**
Foundation Stage •Use developing mathematical, ideas and methods to solve practical problems • Match sets of objects to numerals that represent the number of objects • Sort objects, making choices and justifying decisions • Talk about, recognise and recreate simple patterns • Describe solutions to practical problems, drawing on experience, talking about own ideas, methods and choices	**Foundation Stage** • Begin to relate addition to combining two groups of objects and subtraction to ‘taking away’ • In practical activities and discussion begin to use the vocabulary involved in adding and subtracting • Count repeated groups of the same size • Share objects into equal groups and count how many in each group

Using and applying mathematics	Calculating
Year 1 • Solve problems involving counting, adding, subtracting, doubling or halving in the context of numbers, measures or money, for example to 'pay' and 'give change' • Describe a puzzle or problem using numbers, practical materials and diagrams; use these to solve the problem and set the solution in the original context	**Year 1** • Relate addition to counting on; recognise that addition can be done in any order; use practical and informal written methods to support the addition of a one-digit number or a multiple of 10 to a one-digit or two-digit number • Understand subtraction as 'take away' and find a 'difference' by counting up; use practical and informal written methods to support the subtraction of a one-digit number from a one-digit or two-digit number and a multiple of 10 from a two-digit number • Use the vocabulary related to addition and subtraction and symbols to describe and record addition and subtraction number sentences
Year 2 • Solve problems involving addition, subtraction, multiplication or division in contexts of numbers, measures or pounds and pence • Identify and record the information or calculation needed to solve a puzzle or problem; carry out the steps or calculations and check the solution in the context of the problem	**Year 2** • Represent repeated addition and arrays as multiplication, and sharing and repeated subtraction (grouping) as division; use practical and informal written methods and related vocabulary to support multiplication and division, including calculations with remainders • Use the symbols +, –, ×, ÷ and = to record and interpret number sentences involving all four operations; calculate the value of an unknown in a number sentence (e.g. ☐ ÷ 2 = 6, 30 – ☐ = 24)
Year 3 • Solve one-step and two-step problems involving numbers, money or measures, including time, choosing and carrying out appropriate calculations • Represent the information in a puzzle or problem using numbers, images or diagrams; use these to find a solution and present it in context, where appropriate using £.p notation or units of measure	**Year 3** • Develop and use written methods to record, support or explain addition and subtraction of two-digit and three-digit numbers • Use practical and informal written methods to multiply and divide two-digit numbers (e.g. 13 × 3, 50 ÷ 4); round remainders up or down, depending on the context • Understand that division is the inverse of multiplication and vice versa; use this to derive and record related multiplication and division number sentences
Year 4 • Solve one-step and two-step problems involving numbers, money or measures, including time; choose and carry out appropriate calculations, using calculator methods where appropriate • Represent a puzzle or problem using number sentences, statements or diagrams; use these to solve the problem; present and interpret the solution in the context of the problem	**Year 4** • Refine and use efficient written methods to add and subtract two-digit and three-digit whole numbers and £.p • Use understanding of place value to multiply and divide whole numbers and decimals by 10, 100 • Develop and use written methods to record, support and explain multiplication and division of two-digit numbers by a one-digit number, including division with remainders (e.g. 15 × 9, 98 ÷ 6) and HTU x U

Using and applying mathematics	Calculating
Year 5 • Solve one-step and two-step problems involving whole numbers and decimals and all four operations, choosing and using appropriate calculation strategies, including calculator use • Represent a puzzle or problem by identifying and recording the information or calculations needed to solve it; find possible solutions and confirm them in the context of the problem	**Year 5** • Use efficient written methods to add and subtract whole numbers and decimals with up to two places • Use understanding of place value to multiply and divide whole numbers and decimals by 10, 100 AND 1000 • Refine and use efficient written methods to multiply and divide, TU × TU, U.t × U and HTU ÷ U
Year 6 • Solve multi-step problems, and problems involving fractions, decimals and percentages; choose and use appropriate calculation strategies at each stage, including calculator use • Represent and interpret sequences, patterns and relationships involving numbers and shapes; suggest and test hypotheses; construct and use simple expressions and formulae in words then symbols (e.g. the cost of c pens at 15 pence each is $15c$ pence)	**Year 6** • Use efficient written methods to add and subtract integers and decimals, to multiply and divide integers and decimals by a one-digit integer, and to multiply two-digit and three-digit integers by a two-digit integer

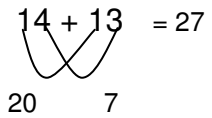
Written methods for addition of whole numbers

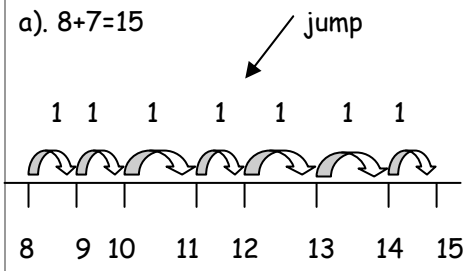
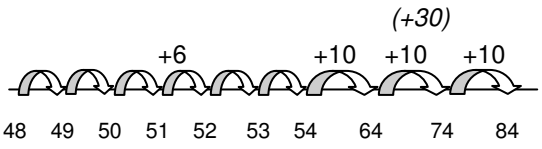
The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for addition which they know they can rely on when mental methods are not appropriate. These notes show the stages in building up to using an efficient written method for addition of whole numbers by the end of Year 4.

To add successfully, children need to be able to:

- recall all addition pairs to 9 + 9 and complements in 10;
- add mentally a series of one-digit numbers, such as 5 + 8 + 4;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.

Stage 1: Partitioning (skipping rope method)	**Stage 1:**
NOTE – before children are ready for this method they will have had a range of experiences including adding groups of objects together practically (using multilink, fingers, real life objects etc) and adding on a number square (adding units then tens) to support their understanding of addition • This is the first step in formal written methods of addition. It introduces the children to the idea of partitioning and that larger numbers are made up of tens and units which can be added together separately. • It builds upon the children's knowledge of numbers and allows them to add in tens and units before partitioning in other ways in the empty number line stage.	Numbers are written along side one another and the "skipping ropes" indicates which part of the number is to be added together (the units and tens) 

Stage 2: The empty number line • The mental methods that lead to column addition generally involve partitioning, e.g. adding the tens and ones separately, often starting with the tens. Children need to be able to partition numbers in ways other than into tens and ones to help them make multiples of ten by adding in steps. • **The empty number line helps to record the steps on the way to calculating the total.**	**Stage 2** Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10. a). 8+7=15  b). 48 + 36 = 84 
Stage 3: Partitioning in columns method • The next stage is to record mental methods using partitioning. Add the tens and then the ones to form partial sums and then add these partial sums. • Partitioning both numbers into tens and ones mirrors the column method where ones are placed under ones and tens under tens. This also links to mental methods.	**Stage 3 – Preferred Method** Partitioned numbers are written under one another: $\begin{array}{rcl} 47 & = & 40 + 7 \\ +\underline{76} & & \underline{70 + 6} \\ & & 110 + 13 = 123 \end{array}$
Stage 4: Expanded column method • The addition of the tens in the calculation 47 + 76 is described in the words 'forty plus seventy equals one hundred and ten', stressing the link to the related fact 'four plus seven equals eleven'. • The expanded method leads children to the more compact method so that they understand its structure and efficiency. <u>The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place value.</u>	**Stage 4** Write the numbers in columns. Adding the units first: $\begin{array}{r} 47 \\ +\ \underline{76} \\ 13 \\ \underline{110} \\ 123 \end{array}$ **<u>Discuss how adding the units first gives the same answer as adding the tens first.</u>**

Stage 5: Column method	Stage 5 – More Able
• In this method, recording is reduced further. Carry digits are recorded below the line, using the words ‘carry ten’ or ‘carry one hundred’, not ‘carry one’. • Later, extend to adding three two-digit numbers, two three-digit numbers and numbers with different numbers of digits.	$\begin{array}{r} 47 \\ +\ 76 \\ \hline 123 \\ {\scriptstyle 1\,1} \end{array}$ $\begin{array}{r} 258 \\ +\ 87 \\ \hline 345 \\ {\scriptstyle 1\,1} \end{array}$ $\begin{array}{r} 366 \\ +458 \\ \hline 824 \\ {\scriptstyle 1\,1} \end{array}$ Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable.

Written methods for subtraction of whole numbers

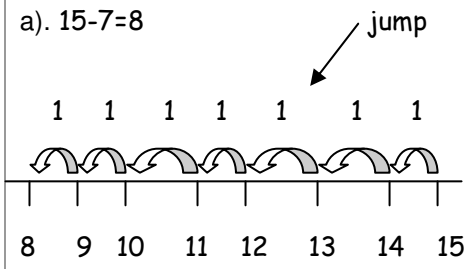
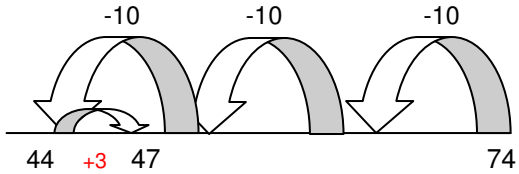
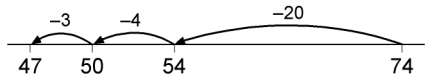
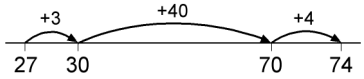
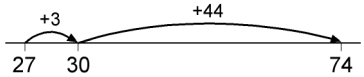
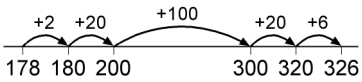
The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for subtraction which they know they can rely on when mental methods are not appropriate.

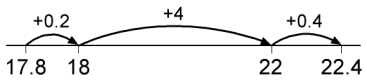
These notes show the stages in building up to using an efficient method for subtraction of two-digit and three-digit whole numbers by the end of Year 4.

To subtract successfully, children need to be able to:

- recall all addition and subtraction facts to 20;
- subtract multiples of 10 (such as 160 – 70) using the related subtraction fact,16 – 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.

Stage 1: Using the empty number line • The empty number line helps to record or explain the steps in mental subtraction. A calculation like 74 – 27 can be recorded by counting back 27 from 74 to reach 47. The empty number line is also a useful way of modelling processes such as bridging through a multiple of ten. • **<u>The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47.</u>** • **<u>With practice, children will need to record less information and decide whether to count back or forward.</u>** It is useful to ask children whether counting up or back is the more efficient for calculations such as 57 – 12, 86 – 77 or 43 – 28. The notes below give more detail on the counting-up method using an empty number line.	**Stage 1** Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10. a). 15-7=8  b).15 – 7 = 8  c). 74 – 27 =  d).74 – 27 = 47 worked by counting back: 
The counting-up method – the difference. • The mental method of counting up from the smaller to the larger number can be recorded using number lines. <u>The number of jumps can be reduced by combining steps.</u> With two-digit numbers, this requires children to be able to work out the answer to a calculation such as 30 + ☐ = 74 mentally.	**Preferred method** a). 74-27=47  b). 74-27=47 
• With three-digit numbers the number of steps can again be reduced, provided that children are able to work out answers to calculations such as 178 + ☐ = 200 and 200 + ☐ = 326 mentally. • The most compact form of recording remains reasonably efficient.	c). 326-178=148 

• The method can be used with decimals where no more than three columns are required. However, it becomes less efficient when more than three columns are needed. • This counting-up method can be a useful alternative for children whose progress is slow, whose mental and written calculation skills are weak and whose projected attainment at the end of Key Stage 2 is towards the lower end of level 4.	d). 22.4-17.8=4.6 
Stage 2: Expanded layout, leading to column method • Partitioning the numbers into tens and ones and writing one under the other mirrors the column method, where ones are placed under ones and tens under tens. • This does not link directly to mental methods of counting back or up but parallels the partitioning method for addition. It also relies on secure mental skills. • The expanded method leads children to the more compact method so that they understand its structure and efficiency. <u>The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and with partitioning.</u>	**Stage 2- More Able** Partitioned numbers are then written under one another: Example: 74 – 27 $\begin{array}{r} 70+4 \\ -\ 20+7 \\ \hline \end{array}$ $\begin{array}{r} \overset{60}{\cancel{70}}+\overset{14}{\cancel{4}} \\ -\ 20+7 \\ \hline 40+7 \end{array}$ $\begin{array}{r} \overset{6}{\cancel{7}}\ \overset{14}{\cancel{4}} \\ -\ 2\ 7 \\ \hline 4\ 7 \end{array}$

Written methods for multiplication of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for multiplication which they know they can rely on when mental methods are not appropriate.

These notes show the stages in building up to using an efficient method for two-digit by one-digit multiplication by the end of Year 4, two-digit by two-digit multiplication by the end of Year 5, and three-digit by two-digit multiplication by the end of Year 6.

To multiply successfully, children need to be able to:

- recall all multiplication facts to 10 × 10;
- partition number into multiples of one hundred, ten and one;
- work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- add combinations of whole numbers using the column method (see above).

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.

Stage 1: Mental multiplication	**Stage 1** Informal recording in Year 4 might be: Arrays ○○○○○○○ ○○○○○…○○ ○○○○○○○ ○○○○○…○○ ○○○○○○○ ○○○○○…○○ 7x3=21 or 5x3= 15 2x3= 6 15+6=21

Stage 2: The grid method

- As a staging post, an expanded method which uses a grid can be used. This is based on the distributive law and links directly to the mental method. It is an alternative way of recording the same steps.

Stage 2 – Preferred method

a). 38 × 7 = 266

×	30	8	
7	*210*	*56*	= 266

b). 138 × 7 = 266

x	100	30	8	
7	700	*210*	*56*	= 966

```
  700
  210
+  56
  966
```

Stage 3: Expanded short multiplication

- The next step is to represent the method of recording in a column format, but showing the working. Draw attention to the links with the grid method above.
- Children should describe what they do by referring to the actual values of the digits in the columns.
- Most children should be able to use this expanded method for TU × U by the end of Year 4.

Stage 3 – More Able

a).
```
  30 + 8
×      7
    5 6   ( 8×7)
  2 1 0   (30×7)
  2 6 6
```

b).
```
   38
  × 7
   56
  210
  266
```

c).
```
  100 +30 +8
x           7
          5 6     ( 8x7)
        2 1 0     (30x7)
        7 0 0     (100x7)
        9 6 6
```

d).
```
   138
 x   7
    56
   210
   700
   966
```

Stage 4: Short multiplication • The recording is reduced further, with carry digits recorded below the line. • If, after practice, children cannot use the compact method without making errors, they should return to the expanded format of stage 3.	**Stage 4** 38 × 7 266 5 The step here involves adding 210 and 50 mentally with only the 5 in the 50 recorded. This highlights the need for children to be able to add a multiple of 10 to a two-digit or three-digit number mentally before they reach this stage.
Stage 5: Two-digit by two-digit products • Extend to TU × TU, asking children to estimate first. • Start with the grid method. The partial products in each row are added, and then the two sums at the end of each row are added to find the total product. • As in the grid method for TU × U in stage 4, the first column can become an extra top row as a stepping stone to the method below.	**Stage 5 – More Able** 56 × 27 is approximately 60 × 30 = 1800. X 50 6 20 1000 120 1120 7 350 42 392 1512 1
• Reduce the recording, showing the links to the grid method above.	56 × 27 is approximately 60 × 30 = 1800. 5 6 × 2 7 4 2 (6× 7) 3 5 0 (50× 7) 1 2 0 (6×20) 10 0 0 (50×20) 15 1 2 1
• Reduce the recording further. • The carry digits in the partial products of 56 × 20 = 120 and 56 × 7 = 392 are usually carried mentally. • <u>The aim is for most children to use this long multiplication method for TU × TU by the end of Year 5.</u>	56 × 27 is approximately 60 × 30 = 1800. 5 6 x 2 7 3 9 2 (56x 7) 1 1 2 0 (56x20) 1 5 1 2 1

Written methods for division of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for division which they know they can rely on when mental methods are not appropriate.

To divide successfully in their heads, children need to be able to:

- understand and use the vocabulary of division – for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;

- recall multiplication and division facts to 10 × 10, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;
- understand and use multiplication and division as inverse operations.

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.

To carry out written methods of division successful, children also need to be able to:

- understand division as repeated subtraction;
- estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92;
- multiply a two-digit number by a single-digit number mentally;
- subtract numbers using the column method.

Stage 1: Short division of TU ÷ U • The key to the efficiency of chunking lies in the estimate that is made before the chunking starts. • Estimating has two purposes when doing a division: – to help to choose a starting point for the division; – to check the answer after the calculation. – • Children who have a secure knowledge of multiplication facts and place value should be able to move on quickly to the more efficient recording on the right. • <u>For most children this will be at the end of Year 4 or the beginning of Year 5.</u>	**Stage 1 – Preferred Method** a). 3)8 21 = 27 - 30 3×10=30 51 - 30 3×10=30 21 3× 7=21 This is then shortened to: **More Able** b). 3)8 21 = 27 • The accompanying patter is 'How many threes divide into 80 so that the answer is a multiple of 10?' This gives 20 threes or 60, with 20 remaining. We now ask: 'What is 21divided by three?' which gives the answer 7. c). 32 r. 4 6)196 – 180 6×30 16 – 12 6× 2 4 32 Answer: 32 R 4 To find 196 ÷ 6, we start by multiplying 6 by 10, 20, 30, … to find that 6 × 30 = 180 and 6 × 40 = 240. The multiples of 180 and 240 trap the number 196. This tells us that the answer to 196 ÷ 6 is between 30 and 40. Start the division by first subtracting 180, leaving 16, and then subtracting the largest possible multiple of 6, which is 12, leaving 4. The quotient 32 (with a remainder of 4) lies between 30 and 40, as predicted.